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EFFECT OF MAGNETO-HYDRODYNAMICS AND COUPLE STRESS ON CHARACTERISTICS OF SINE CURVE SLIDER BEARINGS

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ABSTRACT

The effect of transverse magnetic field and couple stress on the characteristics of sine curve slider bearings is analysed in this paper. The expressions for pressure, load carrying capacity, frictional force and coefficient of friction are derived. The results are presented for different operating parameters. It is observed that the effect of applied magnetic field is to increase the pressure, load carrying capacity, frictional force and coefficient of friction as compared to the corresponding non-magnetic case, and the effect of couple stress is to increase the pressure, load carrying capacity, frictional force but decreases the coefficient of friction as compared to the corresponding Newtonian case.

Keywords: *Sine curve slider bearing, Couple-stress fluid, Magnetic field.*

I. INTRODUCTION

Magneto hydrodynamics (MHD) is a phenomenon proceeding from the motion of electrically conducting fluids such as plasma or fluid metals in the presence of electric or magnetic fields. Many researchers studied the concept of Magneto hydrodynamics and its applications. Magneto hydrodynamics is used to find the effect of externally applied magnetic field on the performance of the different types of bearings, Magneto-hydrodynamic lubrication of finite slider bearings is analyzed by Lin [1], Hughes [2] studied the Magneto-hydrodynamic finite step slider bearing, and they realized that the application of magnetic field which increases the load carrying capacity and the frictional forces. Maki [3] studied the Magneto-hydrodynamic lubrication flow between parallel plates. Kuzma [4] studied the Magneto-hydrodynamic finite journal bearing, Lin and Lu [5] presented dynamic characteristics of MHD wide slider bearing with an exponential film profile and found that the presence of applied magnetic fields signifies an enhancement of film pressure. Magneto-hydrodynamic slider bearing is studied by Snyder [6] and analyzed that the load carrying capacity of the bearing depends on the electromagnetic boundary condition passing through the conductivity of the surface.

Many researchers used the concept of couple stress theory to study the effects on all kinds of bearings. Wang et al [7] studied the performance of dynamically loaded journal bearings lubricated with couple stress fluids. Yan-Yan Ma at. al. [8] studied a dynamically loaded journal bearings lubricated with Non-Newtonian couple stress fluids, the result concludes that couple stress fluids lubrication improves the bearing performance under dynamic loads. Gupta [12] studied the effect of couple stress on hydrostatic thrust bearing. Several investigators [9-12] studied the lubrication problems with Stokes [13] couple stress fluid theory as lubricant.

Hanumagowda [14] studied the effect of Magnetohydrodynamics and couple stress on steady and dynamic characteristics of plane slider bearing, and analyzed that the MHD and couple stress effect improves the steady state and dynamic stiffness and damping characteristics of the plane slider bearings. Biradar Kashinath [15] studied MHD effect of composite slider bearing lubricated with couple-stress fluids, and they found that the fluid film pressure, load carrying capacity, frictional force and coefficient of friction increases as the strength of the magnetic field increases. Naduvinamani [16] studied the effect of MHD couple stress on dynamic characteristics on exponential slider bearing, and shown higher efficiency for small film thickness at higher value of Hartmann number. The effects of couple stress on porous slider bearings by Bujurke et al [17] and found that the load carrying capacity increases and coefficient of friction decreases. Many researchers [18-20] studied the effect of Magnetohydrodynamics with couple stress on fluids for different types of bearings and plates.

Analysis of MHD effect on sine curved slider bearing lubricated with Non-Newtonian fluid has not been studied so far. In this paper, the Analysis of MHD effect on sine curved slider bearing lubricated with Non-Newtonian fluid is analysed.

The physical configuration of the sine curved slider bearing of length L lubricated with incompressible electrically conducting couple stress fluid in the presence of applied magnetic field is shown in Fig. 1. The lower plate of the bearing slides with velocity Uin X - direction and an external transverse magnetic field B_0 is applied in Y – direction. It is assumed that couple and body forces are absent and the effect of internal forces is negligible compare to viscous terms. Under these conditions, the governing basic equations for the flow of couple stress fluid in the presence of applied magnetic field are reduced.

II. MATHEMATICAL ANALYSIS

The physical configuration of the sine curved slider bearing of length L_1 lubricated with incompressible electrically conducting fluid in the presence of applied magnetic field is shown in Fig. 1. The lower plate of the bearing slides with velocity U in x- direction and an external transverse magnetic field B_0 is applied in y – direction. Under these condition, the governing basic equations for the flow of couple stress fluid in the presence of applied magnetic field reduces to the form,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\mu\frac{\partial^2 u}{\partial y^2}-\eta\frac{\partial^4 u}{\partial y^4}-\sigma B_0^2 u=\frac{\partial p}{\partial x}+\sigma E_z B_0 \tag{2}$$

$$\frac{\partial p}{\partial y}=0 \tag{3}$$

Where u, v are the velocity components in x and y directions respectively, p is the pressure, μ is the fluid viscosity, η is a new material constant responsible for the couple stress fluid.For the sine curved slider bearing, the mathematical expression for film thickness is

$$h=h_1+(h_0-h_1)\left\{1-\sin\left(\frac{\pi x}{2L}\right)\right\}$$

where h_1 is the inlet thickness and h_0 is the minimum thickness at the exit.

The boundary conditions are:

At the upper surfaces ($y = h$)

$$u=0,\qquad \frac{\partial^2 u}{\partial y^2}=0,\qquad v=0 \tag{4}$$

At the lower surfaces ($y = 0$)

$$u=U,\qquad \frac{\partial^2 u}{\partial y^2}=0\qquad v=0 \tag{5}$$

On solving equation (1) and (2) using the boundary condition (4) and (5) we get

$$u=-\frac{U}{2}\xi_1-\frac{h_0^2 h}{2l\mu M_0^2}\frac{\partial p}{\partial x}\xi_2 \tag{6}$$

where

$$\xi_1=\xi_{11}-\xi_{12},\quad \xi_2=\xi_{13}-\xi_{14}\ \text{for } 4M_0^2 l^2/h_0^2<1 \tag{7a}$$

$$\xi_1 = \xi_{21} - \xi_{22},\ \ \xi_2 = \xi_{23} - \xi_{24} \ \text{ for } 4M_0^2 l^2 / h_0^2 = 1 \qquad (7b)$$

$$\xi_1 = \xi_{31} - \xi_{32},\ \ \xi_2 = \xi_{33} - \xi_{34} \ \text{ for } 4M_0^2 l^2 / h_0^2 > 1 \qquad (7c)$$

M_0 denotes the Hartmann number given by $M_0 = B_0 h_0 \left(\sigma/\mu\right)^{1/2}$ and $\eta/\mu = l^2$

The associated relations in equations (7a), (7b) and (7c) are given in Appendix A.

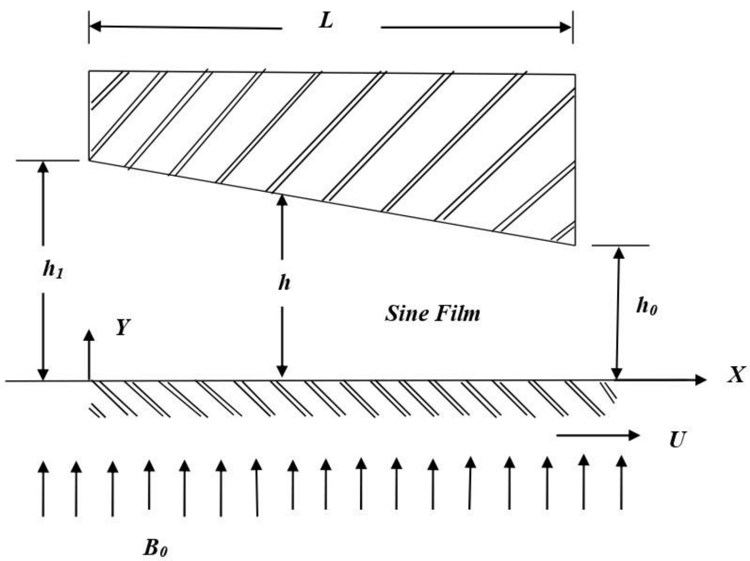

Figure 1: Sine curved slider bearing.

Integration of the continuity equation (1) over the film thickness and the use of boundary conditions (4) and (5) give the Reynolds equation in the form

$$\frac{\partial}{\partial x}\left\{ f(h,l,M_0)\frac{\partial p}{\partial x}\right\} = 6U\frac{dh}{dx} \qquad (8)$$

$$\text{where } f(h,l,M_0) = \begin{cases} \dfrac{6h_0^2 h^2}{\mu l M_0^2}\left\{ \dfrac{A^2 - B^2}{\dfrac{A^2}{B}\tanh\dfrac{Bh}{2l} - \dfrac{B^2}{A}\tanh\dfrac{Ah}{2l}} - \dfrac{2l}{h}\right\} & \text{for } 4M_0^2 l^2/h_0^2 < 1 \\[2ex] \dfrac{6h_0^2 h^2}{\mu l M_0^2}\left\{ \dfrac{2\left(Cosh\left(h/\sqrt{2}l\right)+1\right)}{3\sqrt{2}Sinh\left(h/\sqrt{2}l\right) - h/l} - \dfrac{2l}{h}\right\} & \text{for } 4M_0^2 l^2/h_0^2 = 1 \\[2ex] \dfrac{6h_0^2 h^2}{\mu l M_0^2}\left\{ \dfrac{M_0\left(CosB_1 h + CoshA_1 h\right)}{h_2\left(A_2 SinB_1 h + B_2 SinhA_1 h\right)} - \dfrac{2l}{h}\right\} & \text{for } 4M_0^2 l^2/h_0^2 > 1 \end{cases} \qquad (9)$$

$$A_2 = \left(B_1 - A_1 Cot\theta\right)\ \ B_2 = \left(A_1 + B_1 Cot\theta\right)$$

Introducing non-dimensional quantities

$$x^* = \frac{x}{L},\ \ P^* = \frac{p^* h_0^2}{\mu U L},\ \ l^* = \frac{2l}{h_0},\ \ h^* = \frac{h}{h_0},\ \ M_0 = B_0 h_0 \left(\frac{\sigma}{\mu}\right)^{1/2}\ \ h^* = h_1^* + \left(1 - h_1^*\right)\left\{1 - \sin\left(\frac{\pi x^*}{2}\right)\right\}$$

Substituting above values in equation (8) we get

$$\frac{\partial}{\partial x^*}\left\{ f(h^*,l^*,M_0)\frac{\partial P^*}{\partial x^*}\right\} = 6\frac{dh^*}{dx^*} \quad (10)$$

$$\text{where } f^*(h^*,l^*,M_0) = \begin{cases} \dfrac{12h^{*2}}{l^* M_0^2}\left\{\dfrac{(A^{*2}-B^{*2})}{\dfrac{A^{*2}}{B^*}\tanh\dfrac{B^*h^*}{l^*}-\dfrac{B^{*2}}{A^*}\tanh\dfrac{A^*h^*}{l^*}}-\dfrac{l^*}{h^*}\right\} & \text{for } M_0^2 l^{*2} < 1 \\ \dfrac{12h^{*2}}{l^* M_0^2}\left\{\dfrac{1+Cosh\left(\sqrt{2}h^*/l^*\right)}{\left(3/\sqrt{2}\right)Sinh\left(\sqrt{2}h^*/l^*\right)-\left(h^*/l^*\right)}-\dfrac{l^*}{h^*}\right\} & \text{for } M_0^2 l^{*2} = 1 \\ \dfrac{12h^{*2}}{l^* M_0^2}\left\{\dfrac{M_0\left(CosB_1^*h^*+CoshA_1^*h^*\right)}{A_2^*SinB_1^*h^*+B_2^*SinhA_1^*h^*}-\dfrac{l^*}{h^*}\right\} & \text{for } M_0^2 l^{*2} > 1 \end{cases} \quad (11)$$

$$A^* = \left\{\frac{1+\left(1-l^{*2}M_0^2\right)^{1/2}}{2}\right\}^{1/2},\ B^* = \left\{\frac{1-\left(1-l^{*2}M_0^2\right)^{1/2}}{2}\right\}^{1/2},\ A_1^* = \sqrt{2M_0/l^*}\,Cos\left(\theta^*/2\right),\ B_1^* = \sqrt{2M_0/l^*}\,Sin\left(\theta^*/2\right)$$

$$\theta^* = \tan^{-1}\left(\sqrt{l^{*2}M_0^2-1}\right),\ A_2^* = \left(B_1^*-A_1^*Cot\theta^*\right)\ B_2^* = \left(A_1^*+B_1^*Cot\theta^*\right)$$

The pressure boundary conditions are given by

$$P^* = 0 \text{ at } x^* = 0,1 \quad (12)$$

Integrating equation (10) using the conditions (12) we obtain

$$p^* = 6\int_{x^*=0}^{x^*}\frac{h^*}{f^*(h^*,l^*,M_0)}dx^* + K\int_{x^*=0}^{x^*}\frac{1}{f^*(h^*,l^*,M_0)}dx^* \quad (13)$$

$$\text{where } K = -\frac{6\displaystyle\int_{x^*=0}^{1}\frac{h^*}{f^*(h^*,l^*,M_0)}dx^*}{\displaystyle\int_{x^*=0}^{1}\frac{1}{f^*(h^*,l^*,M_0)}dx^*}$$

The load carrying capacity per unit width is given by

$$w = \int_0^L p\,dx \quad (14)$$

The non-dimensional load carrying capacity is given by

$$W^* = 6\int_0^1\int_{x^*=0}^{x^*}\frac{h^*}{f^*(h^*,l^*,M_0)}dx^*dx^* + K\int_0^1\int_{x^*=0}^{x^*}\frac{1}{f^*(h^*,l^*,M_0)}dx^*dx^* \quad (15)$$

The components of stress tensor required for calculating frictional force is

$$\tau_{yx} = \mu\frac{\partial u}{\partial y} - \eta\frac{\partial^3 u}{\partial y^3}$$

$$\tau_{yx}\big|_{y=0} = -G(h,l,M_0) - \frac{h}{2}\frac{\partial p}{\partial x}$$

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$$\text{where } G(h,l,M_0)=\begin{cases}\dfrac{\mu U l M_0^2}{2h_0^2(A^2-B^2)}\left\{\dfrac{A^2}{B}\coth\left(\dfrac{Bh}{2l}\right)-\dfrac{B^2}{A}\coth\left(\dfrac{Ah}{2l}\right)\right\} & \text{for } 4M_0^2l^2/h_0^2<1\\ \dfrac{\mu U}{16l^2}\left\{\dfrac{h+3\sqrt{2}l Sinh\left(h/\sqrt{2}l\right)}{Cosh\left(h/\sqrt{2}l\right)-1}\right\} & \text{for } 4M_0^2l^2/h_0^2=1\\ \dfrac{\mu U}{2}\left\{\dfrac{(K_1-K_2Cot\theta)\,SinhA_1h+(K_1Cot\theta+K_2)\,SinB_1h}{CoshA_1h-CosB_1h}\right\} & \text{for } 4M_0^2l^2/h_0^2>1\end{cases}$$

$$K_1=\sqrt{M_0/lh_0}\,Cos(\theta/2)\left[1-(lM_0/h_0)\left\{1-4Sin^2(\theta/2)\right\}\right]$$
$$K_2=\sqrt{M_0/lh_0}\,Sin(\theta/2)\left[1+(lM_0/h_0)\left\{1-4Cos^2(\theta/2)\right\}\right]$$

The frictional force is given by

$$F=\int_0^1 (t_{21})_{y=0}\,dx=\int_0^L\left[-G(h,l,M_0)-\frac{h}{2}\frac{\partial p}{\partial x}\right]dx \quad (16)$$

The non-dimensional frictional force is

$$F^*=\int_0^1 G(h^*,l^*,M_0)dx^*+3\int_0^1\left\{\frac{h^*}{\xi(h^*,l^*,M_0)}\right\}dx^*+\frac{K}{2}\int_0^1\left(\frac{1}{\xi(h^*,l^*,M_0)}\right)dx^* \qquad (17)$$

$$\text{where } G^*(h^*,l^*,M_0)=\begin{cases}\dfrac{l^*M_0^2}{24(A^{*2}-B^{*2})}\left(\dfrac{A^{*2}}{B^*}\coth\dfrac{B^*h^*}{l^*}-\dfrac{B^{*2}}{A^*}\coth\dfrac{A^*h^*}{l^*}\right) & \text{for } M_0^2l^{*2}<1\\ \dfrac{1}{48l^{*2}}\left\{\dfrac{2h^*+3\sqrt{2}l^*Sinh\left(\sqrt{2}h^*/l^*\right)}{Cosh\left(\sqrt{2}H/l^*\right)-1}\right\} & \text{for } M_0^2l^{*2}=1\\ \dfrac{(K_1^*-K_2^*Cot\theta^*)\,SinhA_1^*h^*+(K_1^*Cot\theta^*+K_2^*)\,SinB_1^*h^*}{12\left(CoshA_1^*h^*-CosB_1^*h^*\right)} & \text{for } M_0^2l^{*2}>1\end{cases}$$

$$K_1^*=\sqrt{2M_0/l^*}\,Cos(\theta^*/2)\left[1-(l^*M_0/2)\left\{1-4Sin^2(\theta^*/2)\right\}\right]$$
$$K_2^*=\sqrt{2M_0/l^*}\,Sin(\theta^*/2)\left[1+(l^*M_0/2)\left\{1-4Cos^2(\theta^*/2)\right\}\right]$$

$$\xi(h^*,l^*,M_0)=\begin{cases}\dfrac{12h^*}{l^*M_0^2}\left\{\dfrac{(A^{*2}-B^{*2})}{\dfrac{A^{*2}}{B^*}\tanh\dfrac{B^*h^*}{l^*}-\dfrac{B^{*2}}{A^*}\tanh\dfrac{A^*h^*}{l^*}}-\dfrac{l^*}{h^*}\right\} & \text{for } M_0^2l^{*2}<1\\ \dfrac{12h^*}{l^*M_0^2}\left\{\dfrac{1+Cosh\left(\sqrt{2}h^*/l^*\right)}{\left(3/\sqrt{2}\right)Sinh\left(\sqrt{2}h^*/l^*\right)-(h^*/l^*)}-\dfrac{l^*}{h^*}\right\} & \text{for } M_0^2l^{*2}=1\\ \dfrac{12h^*}{l^*M_0^2}\left\{\dfrac{M_0\left(CosB_1^*h^*+CoshA_1^*h^*\right)}{A_2^*SinB_1^*h^*+B_2^*SinhA_1^*h^*}-\dfrac{l^*}{h^*}\right\} & \text{for } M_0^2l^{*2}>1\end{cases}$$

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The coefficient of friction is given by

$$C = \frac{F^*}{W^*} \quad (18)$$

III. RESULTS AND DISCUSSIONS

The effect of Non-dimensional couple stress parameter l^* and Magnetohydrodynamics on Sine film slider bearing is presented in this paper. The parameter M_0 is the Hartmann number and l^* (= $2l / h$) where $l\ (= (\eta / h)^{1/2})$ arises due the presence of small polar additives in the lubricant. The dimension of (η / μ) is of length square and this length may be regarded as chain length of the polar additives in the lubricant. Hence in the present analysis the following range of values of Non-dimensional parameters are used to obtain the bearing characteristics.

- Hartmann number $M_0 = 0 – 3$.
- Couple stress parameter $l^* = 0 – 0.6$.
- Film thickness $h_1^* = 0 – 1.5$

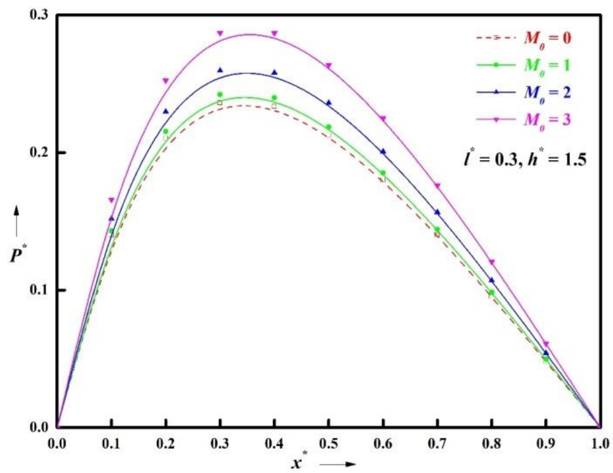

Figure 2: Variation of Non-dimensional pressure P^* with x^* for different values of Hartmann number M_0 and $l^* = 0.3$, h^* = 1.5.

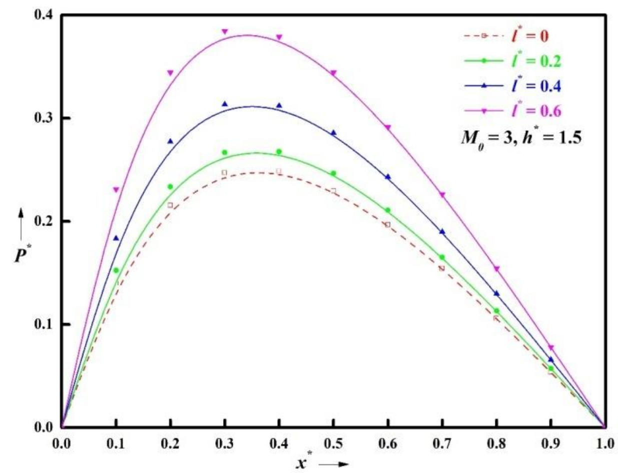

Figure 3: Variation of Non-dimensional pressure P^* with x^* for different values of couple stress parameter l^* and M_0 = 3, h^*= 1.5.

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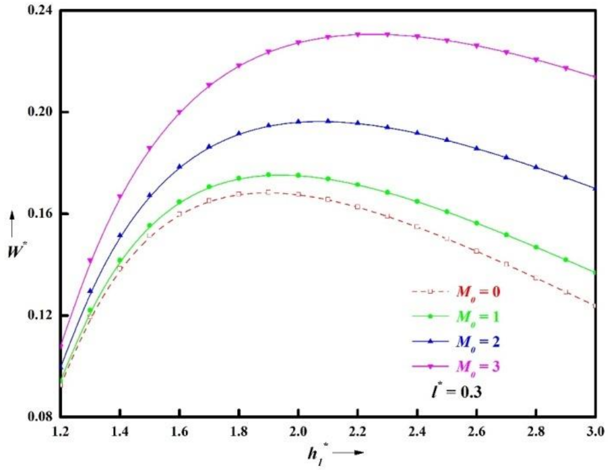

Figure 4: Variation of Non-dimensional load W with h1* for different values of Hartmann Number M0 and l* = 0.3.*

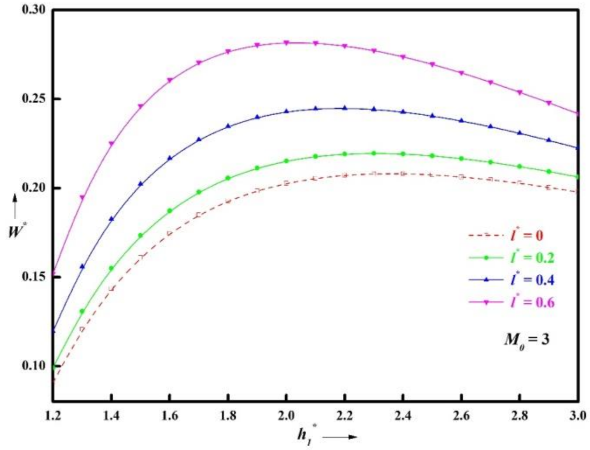

Figure 5: Variation of Non-dimensional load W with h1* for different values of couple stress parameter l* and M0= 3.*

Pressure

Figure 2 depicts the variation of non-dimensional pressure P^* as a function of x^* for different values of Hartman number M_0 with $l^* = 0.3$ and $h_1^* = 1.5$. It is observed that, the effect of magnetic field is to increase the pressure as compared to non-magnetic case (M_0=0). The variation of non-dimensional pressure verses x^* for different values of l^* with $M_0 = 3$ and $h_1^* = 1.5$ is presented in figure 3. It is found that, the effect of couple stress is to increase the pressure as compared to Newtonian case ($l^* = 0$)..

Load carrying capacity

The variation of non-dimensional load carrying capacity W^* with respect to h_1^* for different values of Hartmann number M_0 with $l^* = 0.3$ is shown in figure 4. It is observed that the effect of magnetic field is to increases the load carrying capacity as compared to the corresponding non-magnetic case (M_0=0). The variation of non-dimensional load carrying capacity W^* with respect to h_1^* for different values of couple stress parameters l^* with M_0=3 is shown in figure 5. It is observed that the effect of couple stress is to increases the load carrying capacity as compared to corresponding Newtonian case (l^*=0).

Frictional force

The variation of Non-dimensional frictional force F^* with h_1^* for different values of M_0 is depicted in Figure.6. It is observed that the Non-dimensional frictional force F^* increases with increase in the values of M_0. Figure.7 shows the variation of Non-dimensional frictional force F^* with h_1^* for different values of couple stress parameter l^*. It is observed that the effect of frictional force F^* increases with increase in the values of l^*.

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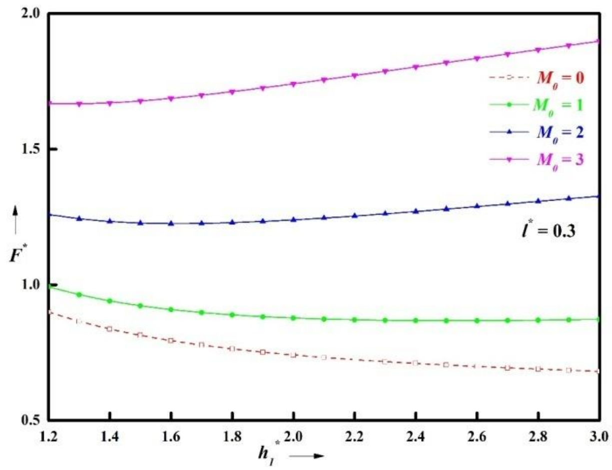

Figure 6: Variation of Non-dimensional Friction F^* with h_1^* for different values of Hartmann Number M_0 and $l^*= 0.3$

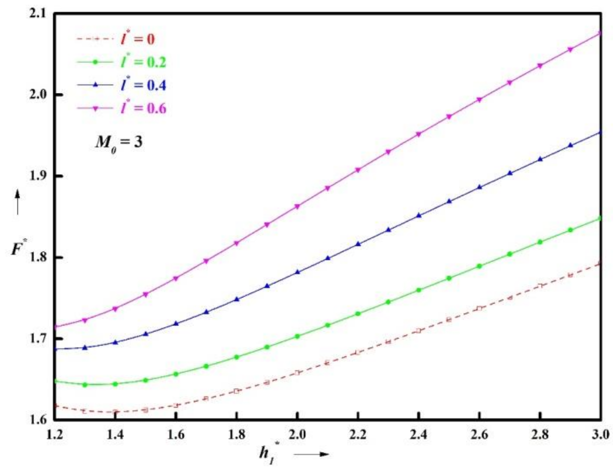

Figure 7: Variation of Non-dimensional Friction F^* with h_1^* for different values of couple stress parameter l^* and $M_0 = 3$.

Coefficient of friction

Figure 7 depicts the variation of coefficient of friction C with h_1^* for different values of M_0.It is observed that, the coefficient of friction C increases with increasing values of M_0 . Figure 8 presents the variation of coefficient of friction verses couple stress parameter l^*. It is observed that coefficient of friction decreases with increasing values couple stress parameter l^*.

IV. CONCLUSIONS

The Analysis of effect of MHD on Sine film Slider Bearings lubricated with Non-Newtonian fluids by using basis of Stokes [13] couple stress fluid theory is presented in this paper

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The following conclusions are drawn from above results,

- The effect of magnetic field is to increase the pressure P^*, load carrying capacity W^* , frictional force F^* and coefficient of friction C as compared to non-magnetic case.
- The effect of couple stress parameter is to increase the pressure P^*, load carrying capacity W^* and frictional force F^*, but coefficient of friction $C$is deceases as compared to Newtonian case.

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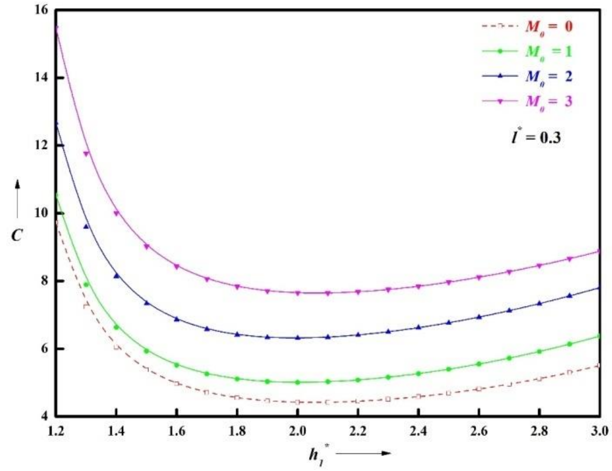

Figure 8: Variation of Non-dimensional Co-efficient of friction C with h_1^* for different values of Hartmann Number M_0 and l^* = 0. 3.

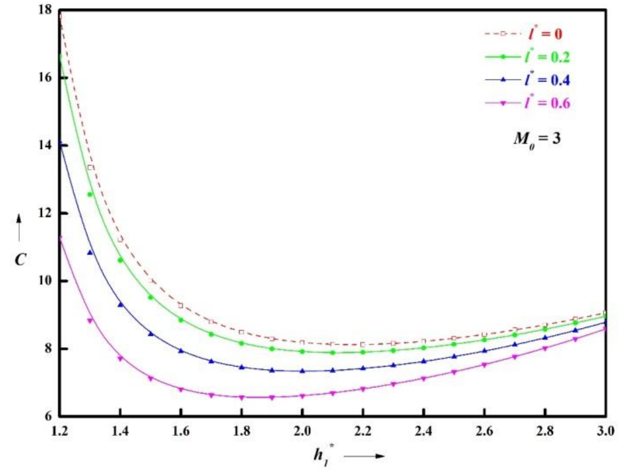

Figure 9: Variation of Non-dimensional Co-efficient of friction C with h_1^* for different values of couple stress parameter l^* and M_0 =3.

Nomenclature

B_0 applied magnetic field
Ccoefficient of friction
F frictional force
F^* non-dimensional frictional force $\left(=-Fh_0/\mu UL\right)$
h film thickness
h_1 inlet film thickness
h_1^* non-dimensional inlet film thickness
h^* non-dimensional film thickness $\left(=h/h_0\right)$
l couple stress parameter $\left(\eta/\mu\right)^{1/2}$
l^* non-dimensional couple stress parameter $\left(2l/h_0\right)$
L Bearing length
M_0 Hartmann number $\left\{=B_0 h_0 (\sigma/\mu)^{1/2}\right\}$
p pressure in the film region
P^* non-dimensional pressure ($= ph_0^2/\mu UL$)
x, y rectangular coordinates
x^* non-dimensional rectangular coordinates $\left(x^* = x/L\right)$
u, v velocity components in film region
w load carrying capacity
W^* non-dimensional load carrying capacity ($= -wh_0^2/\mu UL^2$)
η material constant characterizing couple stress
μ viscosity coefficient
σ electrical conductivity

Appendix A

$$\xi_{11} = \frac{B^2}{(A^2 - B^2)}\left\{\frac{Sinh(Ah/l) - Sinh(Ay/l) - Sinh\,A(h-y)/l}{Sinh(Ah/l)}\right\} \quad \text{(A1a)}$$

$$\xi_{12} = \frac{A^2}{(A^2 - B^2)}\left\{\frac{Sinh(Bh/l) - Sinh(By/l) - Sinh\,B(h-y)/l}{Sinh(Bh/l)}\right\} \quad \text{(A1b)}$$

$$\xi_{13} = \frac{B^2\{Sinh(Ah/l) - Sinh(Ay/l) + Sinh\,A(h-y)/l\}}{Sinh(Ah/l)\{(B^2/A)\tanh(Ah/2l) - (A^2/B)\tanh(Bh/2l)\}} \quad \text{(A1c)}$$

$$\xi_{14} = \frac{A^2\{Sinh(Bh/l) - Sinh(By/l) + Sinh\,B(h-y)/l\}}{Sinh(Bh/l)\{(B^2/A)\tanh(Ah/2l) - (A^2/B)\tanh(Bh/2l)\}} \quad \text{(A1d)}$$

$$A = \left[\frac{1 + \{1 - (4l^2 M_0^2 / h_2^2)\}^{1/2}}{2}\right]^{1/2} \quad B = \left[\frac{1 - \{1 - (4l^2 M_0^2 / h_2^2)\}^{1/2}}{2}\right]^{1/2} \quad \text{(A1e)}$$

$$\xi_{21} = \frac{Sinh\{(y-h)/\sqrt{2}l\} + Sinh(y/\sqrt{2}l) - Sinh(h/\sqrt{2}l)}{Sinh(h/\sqrt{2}l)} \quad \text{(A2a)}$$

$$\xi_{22} = \frac{yCosh\{(y-h)/\sqrt{2}l\} + yCosh(y/\sqrt{2}l) - hCosh(h/\sqrt{2}l) - h}{2\sqrt{2}lSinh(h/\sqrt{2}l)} \quad \text{(A2b)}$$

$$\xi_{23} = \frac{ySinh\{(y-h)/\sqrt{2}l\} + ySinh(y/\sqrt{2}l) - hSinh(y/\sqrt{2}l)}{6lSinh(h/\sqrt{2}l) - \sqrt{2}h} \quad \text{(A2c)}$$

$$\xi_{24} = \frac{2Cosh\{(y-h)/\sqrt{2}l\} + 2Cosh(y/\sqrt{2}l) - 2Cosh(h/\sqrt{2}l) - 2}{3\sqrt{2}Sinh(h/\sqrt{2}l) - (h/l)} \quad \text{(A2d)}$$

$$\xi_{31} = \frac{CoshA_1 yCosB_1(y-h) - CosB_1 yCoshA_1(y-h)}{(CoshA_1 h - CosB_1 h)} \quad \text{(A3a)}$$

$$\xi_{32} = \frac{Cot\theta\{SinhA_1 ySinB_1(y-h) - SinB_1 ySinhA_1(y-h)\} + (CoshA_1 h - CosB_1 h)}{(CoshA_1 h - CosB_1 h)} \quad \text{(A3b)}$$

$$\xi_{33} = \frac{Cot\theta\{SinB_1 ySinhA_1(y-h) + SinhA_1 ySinB_1(y-h)\} + (CosB_1 h + CoshA_1 h)}{(B_1 - A_1 Cot\theta)SinB_1 h + (A_1 + B_1 Cot\theta)SinhA_1 h} \quad \text{(A3c)}$$

$$\xi_{34} = \frac{CosB_1 yCoshA_1(y-h) + CoshA_1 yCosB_1(y-h)}{(B_1 - A_1 Cot\theta)SinB_1 h + (A_1 + B_1 Cot\theta)SinhA_1 h} \quad \text{(A3d)}$$

$$A_1 = \sqrt{M_0/lh_0}\,Cos(\theta/2) \quad B_1 = \sqrt{M_0/lh_0}\,Sin(\theta/2) \quad \theta = \tan^{-1}\left(\sqrt{4l^2 M_0^2 / h_0^2 - 1}\right) \quad \text{(A3e)}$$

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